



Dependence of the confinement time of an electron plasma on the magnetic field in a quadrupole Penning trap

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Abstract

A quadrupole Penning trap is used to confine electrons in weak magnetic fields. Perturbations due to space charge and imperfections in the trap geometry, as well as collisions with the background gas molecules, lead to loss of the electrons from the trap. We present in this work the results on measurements of the electron confinement time and its dependence on the magnetic field in a quadrupolar Penning trap. We describe a method to measure the confinement time of an electron cloud under weak magnetic fields (0.01 T - 0.1 T). This time is found to scale as $\tau \propto B^{1.41}$ in variance with the theoretically expected confinement time that scales as $\tau \propto B^2$ for trapped electrons that are lost through collisions with the neutrals present in the trap. A measurement of the expansion rate of the electron plasma in the trap through controlled variation of the trap voltage, yields expansion times that depend on the energy of escaping electrons. This is found to vary in our case in the scaling range $B^{0.32}$ to $B^{0.43}$. Distorting the geometry of the trap, results in a marked change in the confinement time's dependence on the magnetic field. The results indicate that the confinement time of the electron cloud in the trap is limited by both, effects of collisions and perturbations that result in the plasma loss through expansion in the trap.

Keywords: Non-neutral plasma, Confinement time, Anharmonicity, Quadrupole penning trap

Introduction

An essential requirement of ion traps is to confine electrons or ions for longer periods of time under minimal perturbations, in order to carry out studies on them [1–4]. However, the confinement time is finite due to several reasons that include collisions with the surrounding neutrals, anharmonicities in the trap potential induced by deviations from the ideal geometry of the trap, and in situations involving a large number of charged particles, space charge effects in the trap [5]. Inhomogeneity in the external magnetic fields in case of Penning traps is an additional perturber and contributes to loss of ions [1, 6]. Confinement times of long durations are realized for high precision measurements after incorporating techniques that restrict these perturbations to minimum levels [2, 7, 8]. In a series of pioneering experiments [9, 10] electron confinement in cylindrical Penning traps (CPT) have been investigated. According to a confinement theorem [11–14], an azimuthally symmetric trap cannot exert a net torque on the trapped electron

cloud and the total angular momentum must remain a constant. This limits the radial expansion of the electron cloud. Furthermore, the electrons or ions cannot escape in the axial direction due to the harmonic potential minimum. Notwithstanding the restrictions on losses imposed by the confinement theorem, losses do occur due to the reasons cited earlier.

In measuring the confinement time of an electron plasma in a CPT it is shown that the confinement time departs from the expected B^2 dependence at low pressures and is governed by other loss mechanisms apart from elastic binary collisions with background neutrals present [9, 14]. Induced asymmetries have been applied to investigate plasma confinement time [10] and these lead to a different dependence on the magnetic field. In work examining the background pressure's effect in addition to the applied magnetic field, B , measurements of the variation of the radial density of trapped electrons at the centre of the trap [12, 15] reveal that the confinement time varies with B and pressure, P , as $\tau \propto B^{3/2}/P$. In CPTs asymmetries induced by application of patch potentials limit the confinement time and the latter has been studied as a function of plasma column length and applied magnetic field [16]. Stoneking et. al, have used $\mathbf{E} \times \mathbf{B}$ drifts to establish equilibrium in a purely toroidal electron plasma and confinement times of ~ 3 s have been obtained [17].

In this work, we carry out investigations of the dependence of the confinement time of an electron plasma on the external magnetic field in a different trap geometry, namely, that of the Quadrupole Penning Trap (QPT). The CPT geometry and electrode structure allows for trapping higher densities of plasma and combination of oppositely charged species, as in anti hydrogen production [18, 19]. The QPT provides for greater stability when fewer particles are trapped and thus has seen wide applications in tests of QED involving single ions [1, 13, 14, 20]. Unlike in the CPT, the QPT has electrodes that are hyperboloids of revolution. When a larger number of ions are trapped in a QPT, such as in our situation with electrons, it is of interest to examine the confinement time's dependence on B and whether the $B^{3/2}$ dependence as seen in CPTs is reproducible in a QPT. Our measurements yield a confinement time, $\tau \propto B^{1.41}$. This is a departure from the expected scaling $\tau \propto B^2$ obtained taking into account collisional interactions alone with the external neutrals in the trap [9, 10, 16], and differs from the scaling law as in CPTs. As a complement to these studies, we have also measured the expansion rate of the plasma cloud, through a controlled variation of the trap potential. These measurements point to the influence of collisions as well as plasma diffusion in determining the dependence of τ on B . We have also carried out measurements with a trap where the internal geometry is altered. This sharply modifies the confinement time's dependence on B suggesting that anharmonicities induced by geometric distortions play a significant role in determining the scaling law.

The paper is organized as follows: After a discussion of the basic theory governing single particle motion, we present a heuristic argument that establishes the dependence of τ on B in the presence of collisions with background gases in the trap. We then present a technique based on LabVIEW for measuring the same. This is followed by results from a technique that estimates the plasma expansion time and the dependence on the magnetic field of this expansion. Further, we have carried out measurements of τ 's dependence on B when the internal geometry of the trap is perturbed. We summarize the results and draw conclusions based on the results.

Theory

Penning traps are widely used as devices for confining non-neutral plasmas and fabricated in many geometrical shapes. An abundance of literature exists on Penning traps and their applications for different geometries [21–23]. We briefly summarize the principle of single charged particle confinement in a Penning trap [1]: A quadrupole Penning trap confines charged particles through combining a static electric potential with a uniform magnetic field B along the symmetry axis of the trap. The quadrupole potential governing single particle motion is described by

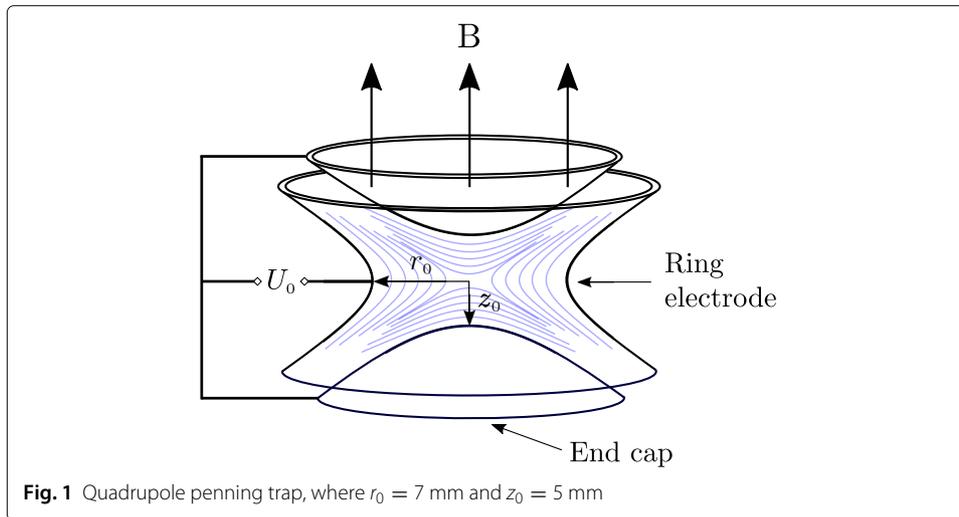
$$\Phi(x, y, z) = \frac{U_0}{r_0^2 + 2z_0^2} (2z^2 - x^2 - y^2) \quad (1)$$

Where U_0 is the applied potential; r and z are the radial and axial coordinates respectively; and d is the characteristic dimension of the trap given by $d^2 = r_0^2 + 2z_0^2$, r_0 being the radius of the ring electrode and z_0 , is half the distance between the two end cap electrodes. Figure 1 shows the electric and magnetic field lines in the trap. The degrees of freedom in the x and y directions (radial motion) are coupled, resulting in coupled equations of motion of the electron. On the other hand, motion of the electron along the z -direction (axial motion) is uncoupled. The solutions of the equations are obtained by a frame transformation yielding the characteristic frequencies ($\omega_z, \omega'_c, \omega_m$) of motion for a single ion or electron [1, 2], where ω_z is the axial oscillation frequency of motion due to the harmonic potential, ω'_c is the modified cyclotron frequency. ω_m is the magnetron frequency representing a drift in the radial plane around the centre of the trap. Note that the axial oscillation has no dependence on the magnetic field and for stable solutions we require that $\omega_c \geq \sqrt{2}\omega_z$, ω_c being the cyclotron frequency. In a QPT collisions with the background neutrals lead to instabilities and an expansion in the radial plane (Fig. 2). In a real trap with a cloud of electrons the radial and axial motion of the electrons are coupled due to Coulomb interactions as well as perturbations in the trap potential that originate from imperfections in the trap geometry. This leads to the presence of higher order frequencies measured as the motional resonances, in addition to the single particle frequencies described here [1, 2, 24, 25]. The trapped electrons with temperature corresponding to a few eV, constitutes a non-neutral weakly coupled plasma with Coulomb coupling parameter $\sim 10^{-4}$, number density $\sim 10^{+15} \text{ m}^{-3}$, Debye length $\sim 0.1 \text{ mm}$ and plasma cloud of radius $\sim 4 \text{ mm}$. Thus, an independent particle model can be employed in our trapped electron plasma conditions for analysis.

Since the kinetic energy of trapped electrons is a few eV and comparable to the typical trap depths which is also a few eV, this leads to a high rate of loss of electrons [26]. The hotter electrons collide more frequently with the background molecules of the gas in the trap and consequently, are lost more easily than the colder electrons. Theoretical calculations based on transport theories [27] in the case of Malmberg Penning traps reveal that collisions cause plasma expansion and the rate is proportional to B^{-2} . We present here a simple argument to illustrate the dependence of the confinement time on the applied magnetic field. The linear frequency of cyclotron motion may be written as

$$f_c = \frac{qB}{2\pi m} \quad (2)$$

$$f_c \propto B \quad (3)$$



The expression for the angular frequency of magnetron motion is given by

$$\omega_m = \frac{\omega_c}{2} - \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}} \tag{4}$$

Since $\omega_z \ll \omega_c$ we can write up to first order,

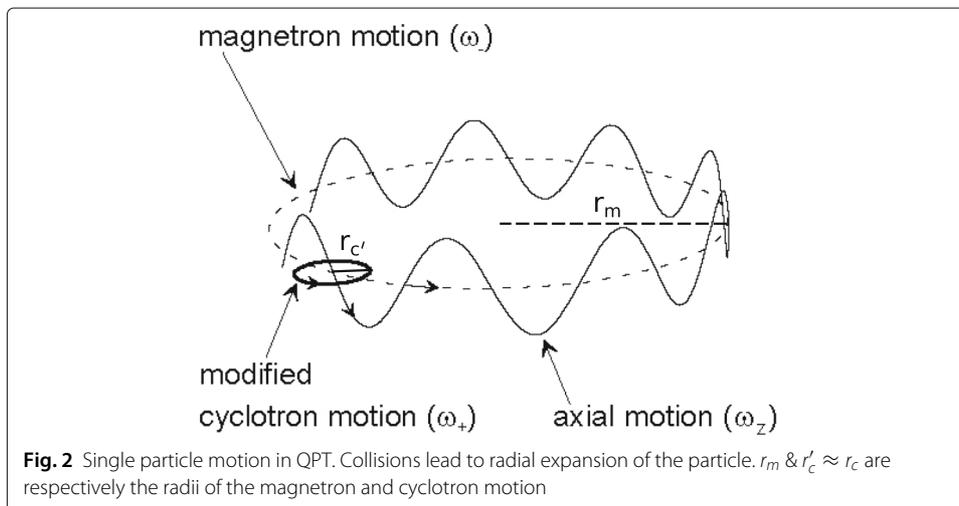
$$\omega_m \approx \frac{\omega_z^2}{2\omega_c} \tag{5}$$

Therefore the linear frequency of magnetron motion

$$f_m \propto \frac{1}{B} \tag{6}$$

Increasing the magnetic field thus, increases the cyclotron frequency and decreases the magnetron frequency. As the magnetic field increases, the radius of the cyclotron orbit decreases due the conservation of angular momentum. Therefore,

$$B \propto \frac{1}{r_c} \tag{7}$$



The frequency of collisions, f , of a single electron in a cycle of the magnetron motion,

$$f \propto r_c r_m \quad (8)$$

where r_c and r_m are respectively, the radii corresponding to the (modified) cyclotron and magnetron motions (Fig. 2).

The energy associated with the single particle motion is given by [28, 29]

$$H = \frac{1}{2}m\omega_1\omega_c r_c^2 - \frac{1}{2}m\omega_1\omega_m r_m^2 + \frac{1}{2}m\omega_1\omega_z^2 r_z^2 \quad (9)$$

where $\omega_1 = \sqrt{\omega_c^2 - 2\omega_z^2}$. Note that the contribution to the energy of the particle due to the magnetron motion is negative, implying that collisions will increase r_m resulting in particle loss [30, 31]. In order to reduce the second term in Eq.9, ω_m has to be decreased (Eq.6), by increasing B.

Thus,

$$f \propto \frac{1}{f_c} \times f_m \quad (10)$$

or,

$$\tau \propto \frac{1}{f} \propto \frac{f_c}{f_m} \propto B^2 \quad (11)$$

Thus, collisions alone should limit the confinement time in a manner that determines the variation of the confinement time as illustrated. However as mentioned earlier, a cloud of electron plasma is subject to loss due to expansion of the cloud on account of trap conditions that cause instabilities [9–16]. The total canonical angular momentum must be conserved, therefore the like-particle interactions which are complex and non-linear cannot expand the plasma radially. Torques may arise from collisions with neutral atoms, angular asymmetries in magnetic field or the containment vessel may act to change the total canonical angular momentum and expand the plasma radially.

Experimental details

Measurement of confinement time

Detection of the trapped electrons is through an electronic tank circuit weakly coupled to the trap. The details of this technique may be found in previous work [1, 24, 25, 32]. The electrons generated from a thoriated tungsten filament are loaded into the trap and the filament current is then switched off while simultaneously setting the storage voltage, U_0 , that is applied between the end cap and ring electrodes. After a fixed duration of 10 ms, starting from the moment the loading of electrons stops, U_0 is ramped down (Fig. 3a and c). This duration is chosen to be greater than the time for electrons to achieve equilibrium (10 μ s) [33]. At the voltage value where the corresponding axial frequency coincides with the tank circuit frequency, resonant energy transfer from the circuit to the oscillating electrons results in a dip in the demodulated DC output of the circuit. The extent of the dip is proportional to the number of electrons axially oscillating, for weak excitation of the circuit oscillation. This sequence is then repeated for different time steps. We thus obtain the loss curve of the trapped electrons as a function of time. However the filament current fluctuates, and hence identical loading of electrons into trap in each trial is not assured, apart from the situation here where each timing and detection cycle results in only a single data point in this case. In a different approach (Fig. 3c), the ramp cycle

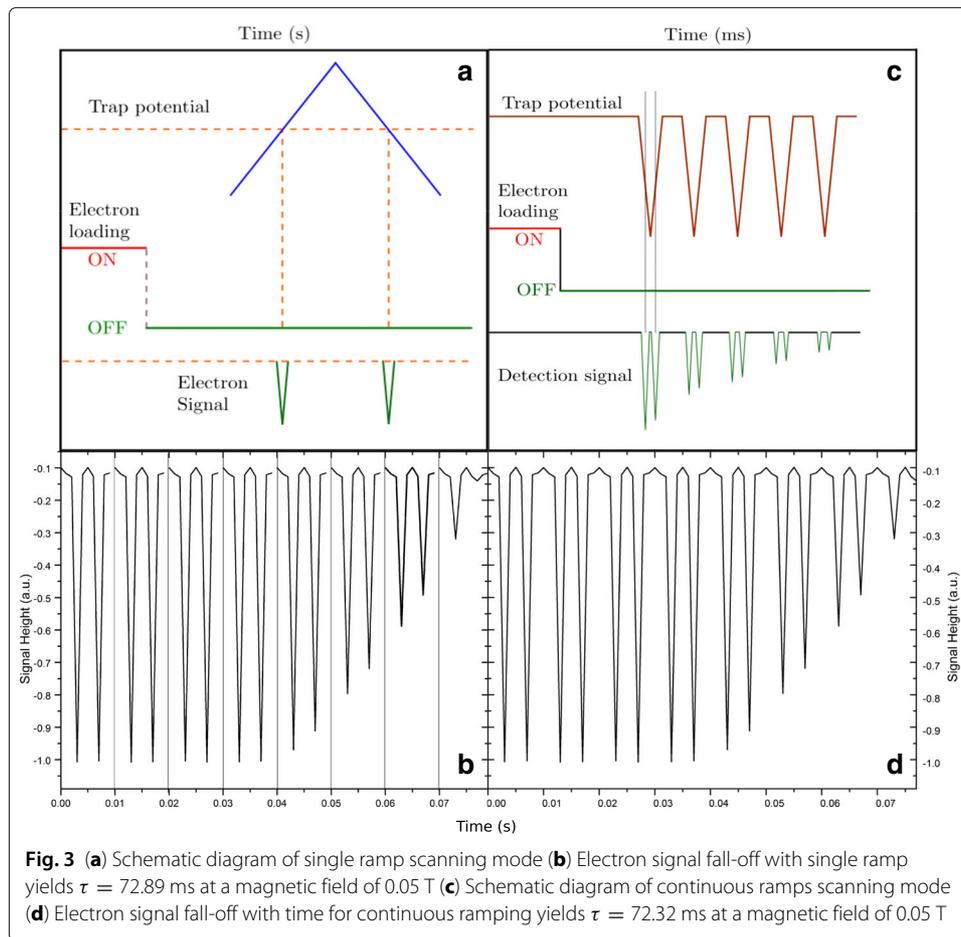


Fig. 3 (a) Schematic diagram of single ramp scanning mode (b) Electron signal fall-off with single ramp yields $\tau = 72.89$ ms at a magnetic field of 0.05 T (c) Schematic diagram of continuous ramps scanning mode (d) Electron signal fall-off with time for continuous ramping yields $\tau = 72.32$ ms at a magnetic field of 0.05 T

is continuously repeated at the same ramp frequency soon after loading of electrons is stopped. This results in the collection of a larger set of data points for a single loading event and excludes any variations induced by possible differences in the electron loading. The confinement time of trapped electrons is measured by acquiring the resonance absorption signal. The resonance absorption signal of the trapped electrons was acquired at fixed intervals of time.

The trapped electrons are continuously sampled over time steps of 10 ms. The disadvantage in this manner of probing for stored electrons is in the repetitive detection cycles that may result in perturbative losses due to the excitation from the coupled tank circuit, weak as it may be, overriding the intrinsic loss mechanisms. However, a comparison of data obtained through both methods yielded no significant differences in measurement of the signal fall off features as shown in Fig. 3b and d. Hence, the latter method is adopted as it yields more data points for a single loading detection event. The area under the signal is plotted against the time interval. The measurement of confinement time by the single ramp as well as continuous ramping yield almost the same values, as shown in Fig. 3b and d.

The magnetic field at the position of the trap between the poles of the electromagnet was obtained through a measurement of the cyclotron frequency of the electrons in the

trap by excitation of the motional spectrum through external RF excitation. The procedure for obtaining the motional spectrum is described elsewhere and we will not elaborate this here [24, 25, 32]. The magnetic field obtained by measurement of the cyclotron frequency is plotted against the current in the electromagnet for a few measurements. This yields the magnetic field at the trap center for all currents (Fig. 4). The dimensions of the trap extend 7 mm radially and 5 mm axially from the trap center. The magnetic field is measured using a hall probe along the X , Y and Z axes from the centre of the pole pieces of the electromagnet. The magnetic field can be considered to be homogeneous within the dimensions of the trap, within a small variation. Figure 5 shows the magnetic field's variation in space around the center of the pole pieces of electromagnet. The variation in any axis was found to be less than 0.6%. The background pressure in the trap was 2×10^{-8} torr, for all measurements.

Measurement of expansion rate of the electron plasma

The trapped non neutral plasma undergoes losses both through collisions and torques induced on the plasma cloud due to anharmonicities present in the trap. As the confinement time is limited by both, collisions with background neutrals, as well as by the inherent instability of the electron plasma, we measure the expansion rate of the plasma as a function of the magnetic field as described in the following section [12, 15, 27, 34]:

The trap voltage is set at a higher value such as 10V to ensure that the entire energy spectrum of electrons is confined by the potential. Loading of the electrons is stopped

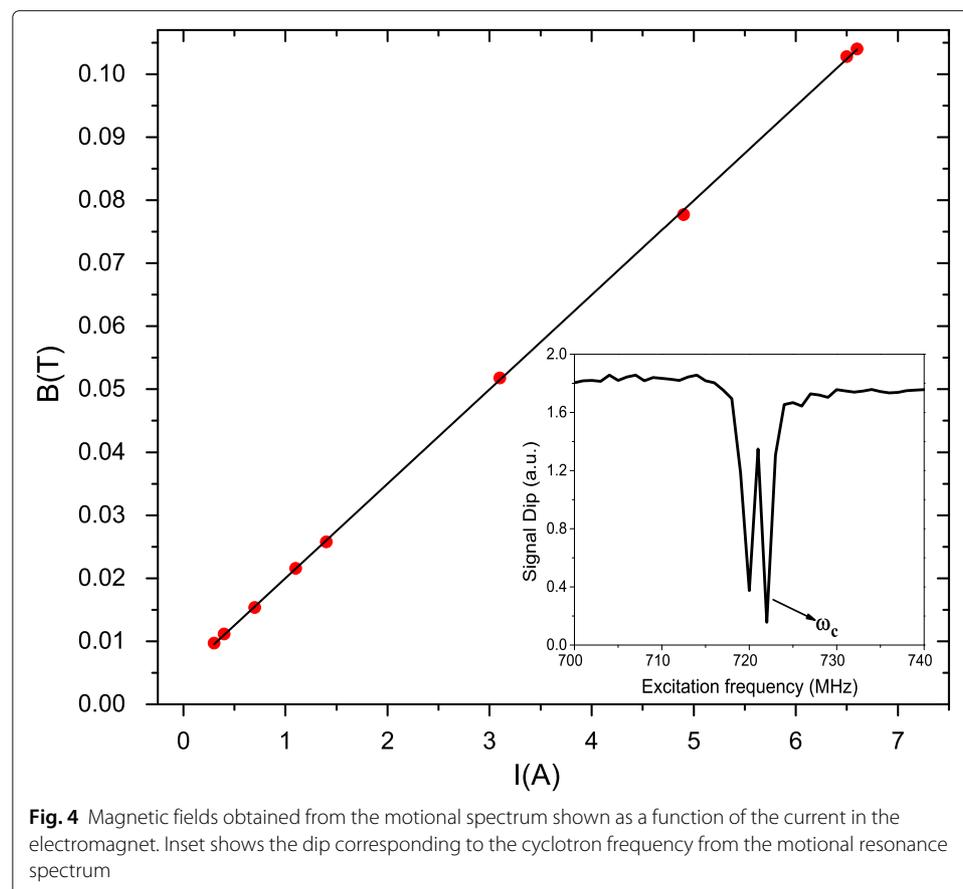
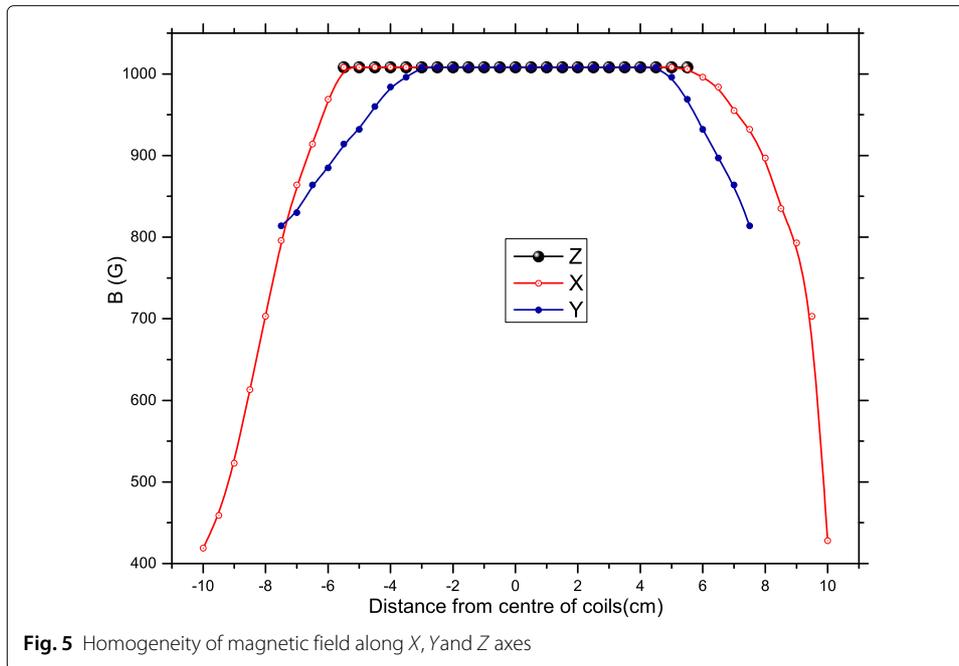


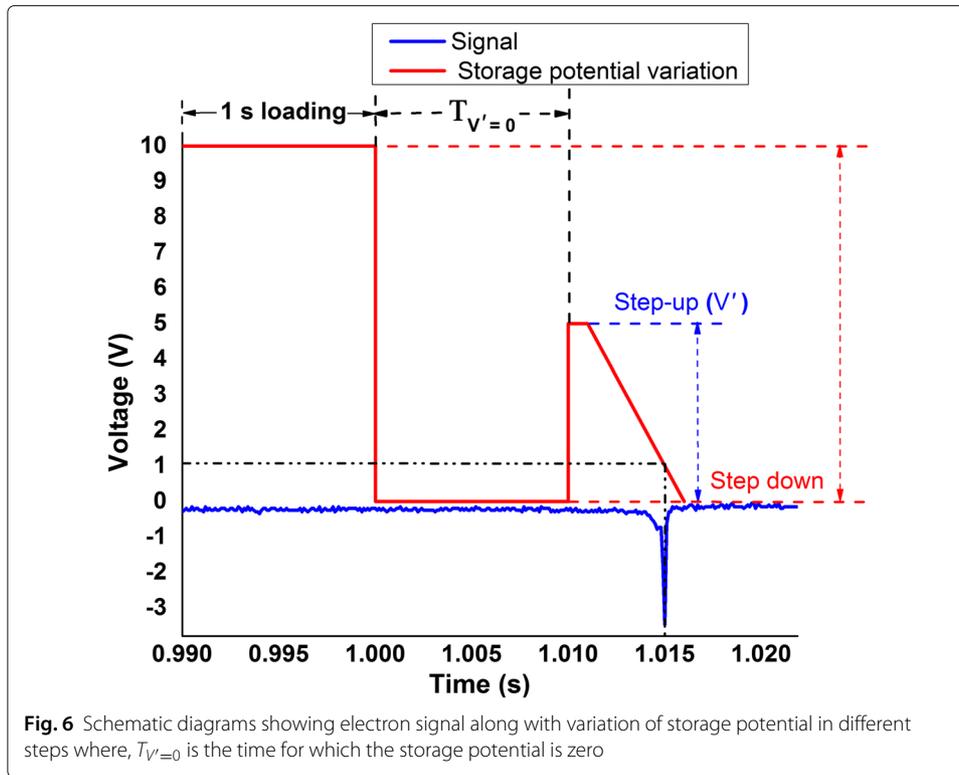
Fig. 4 Magnetic fields obtained from the motional spectrum shown as a function of the current in the electromagnet. Inset shows the dip corresponding to the cyclotron frequency from the motional resonance spectrum



and simultaneously the storage voltage is reduced to zero. This results in the escape of electrons, with electrons energies greater than eV exiting the trapping region faster than the lower energy (less than eV) electrons that are confined within the potential. After a certain time, the voltage is ramped up rapidly to a certain voltage V' . This results in recapture of the electrons that are still in the trapping region with energy range around V' . The schematic of this sequence is shown in Fig. 6 and shows results of the dwell time measurements before stepping up V' to different levels. The time increases as the step up voltage, as expected, since higher step up voltages, results in the recapture of a wider energy range of electrons, whereas lower step up voltages result in capturing only the electrons of energies less than the range corresponding to the lower voltage.

As the axial energy distribution measurements we have carried out indicate, the electron energies are centered around 5V, the bias on the electron source [33]. This measurement also points to the dependence of the electron cloud expansion rate on the energy range of the electrons, i.e. diffusion of lower energy electrons from the centre of trap is slower than their higher energy counterparts.

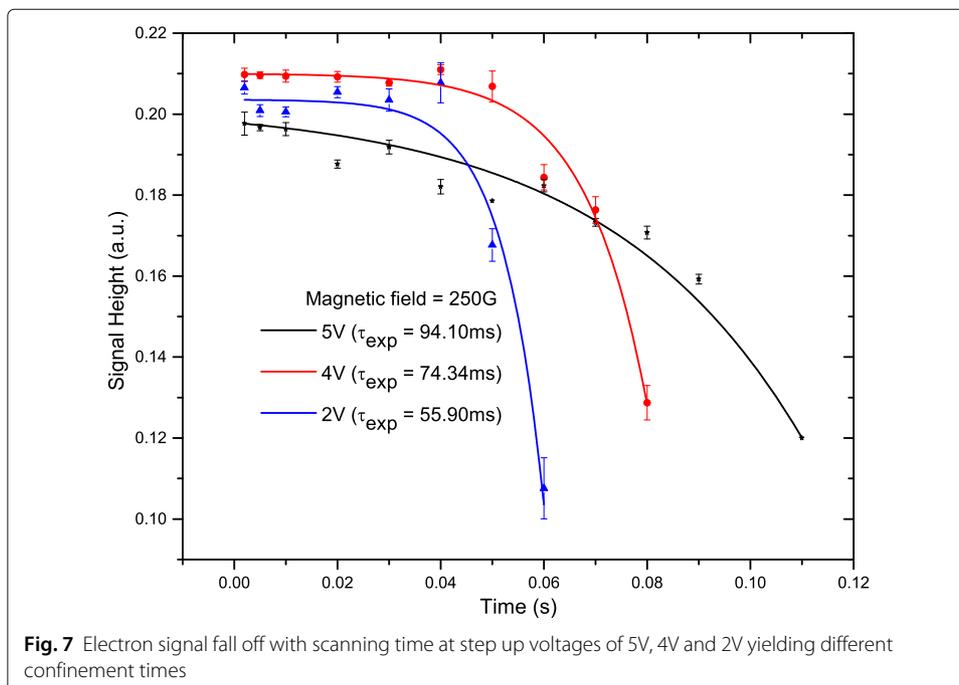
The experiment was performed at different magnetic fields and for different times, until the signal cannot be detected at any step up voltage, V' . The signal decay versus time for different magnetic fields is shown in the Fig. 7 and shows the expansion rate for different magnetic fields for step up voltages of 2V, 4V and 5V. One can note that in all situations measuring the electron loss as a function of time, the signal is initially constant and then sharply drops. The absence of an exponential fall off pattern throughout the measurement time, we conjecture, has to do with the electrons gaining velocity and hence collision frequency as they diffuse out of the trap. The trapped non neutral plasma undergoes losses both through collisions and torques induced on the plasma cloud due to anharmonicities present in the trap. The latter results in the expansion of the plasma [12, 15, 26, 27].

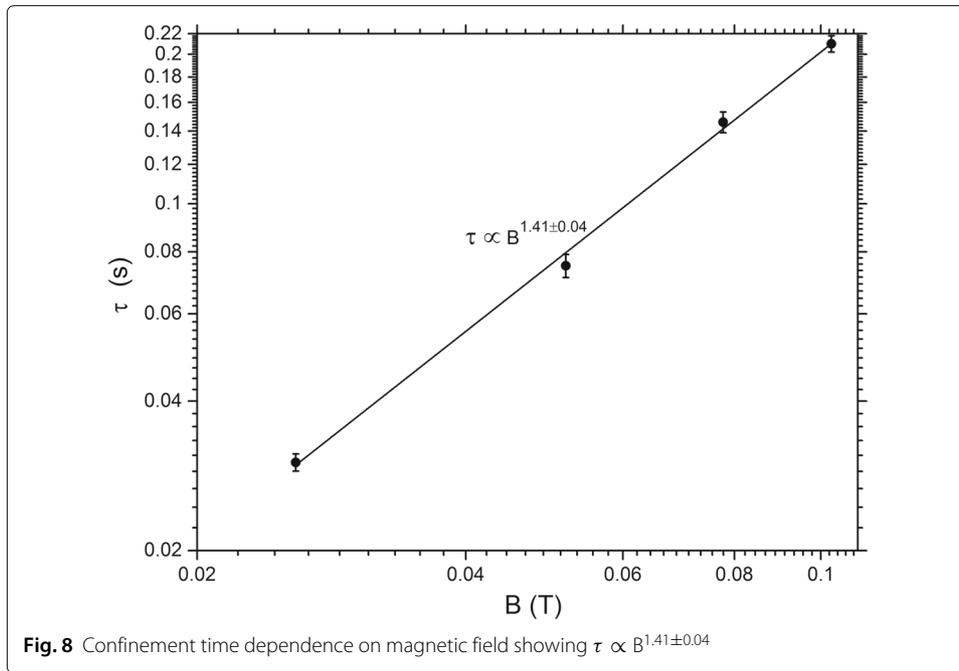


Results and discussion

Dependence on magnetic field

Figure 3b shows the decay of signal area with dwell time recorded in the single ramp mode. All data was recorded at the lowest pressure attainable under normal operating conditions of our setup (low 10^{-8} Torr). Figure 8 shows the dependence of B on τ revealing that $\tau \propto B^{1.41 \pm 0.04}$.





Plasma expansion rate dependence on magnetic field

When the storage potential is made zero, the plasma expands and is lost from the trap in the axial direction primarily, although the nonlinear couplings in the degrees of freedom result in radial expansion as well. The coupling between the radial and axial degrees of freedom leads to a dependence on the magnetic field of the plasma expanding primarily in the axial direction, in this set of experiments.

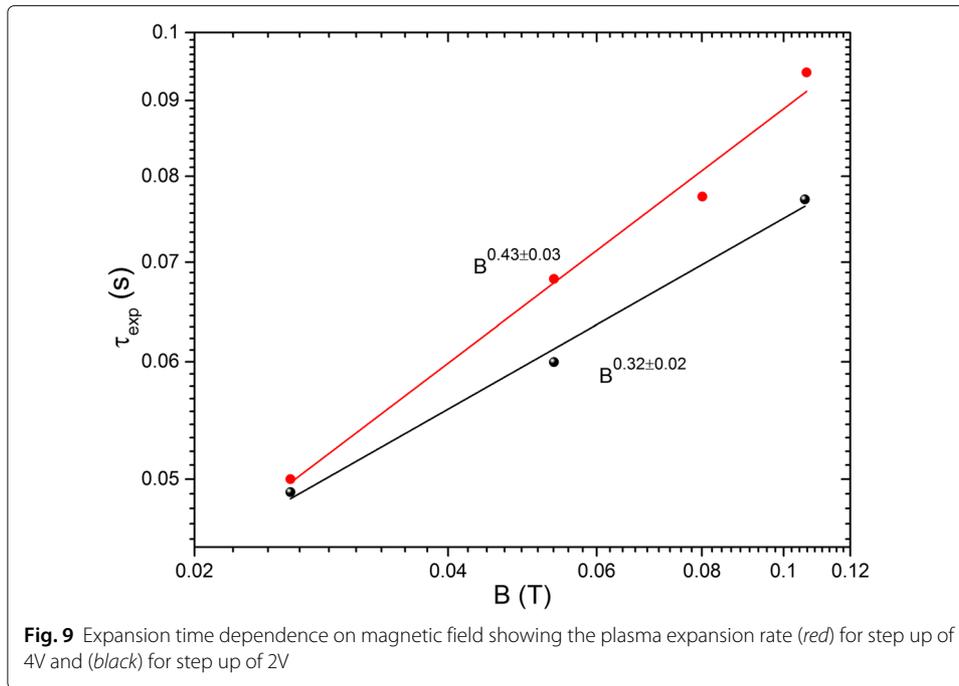
The slope of the plot of $B - \tau$ on a logarithmic scale yield slopes of 0.32 ± 0.02 and 0.43 ± 0.04 , for step up voltages of 2V and 4V respectively, as shown in Fig. 9. Therefore $B - \tau$ graphs show that the plasma expansion time depends on magnetic field in the range $B^{0.32 \pm 0.02} - B^{0.43 \pm 0.04}$, indicating the dependence of this expansion on the kinetic energy range of the electrons.

For a strongly magnetized plasma $\omega_{ce}^2 \gg \omega_{pe}^2$, where ω_{ce} and ω_{pe} are the cyclotron frequency and the plasma frequency of electron plasma respectively [11] and the plasma expands on a time scale given by the diffusion time, $\tau_{diff} \sim (\omega_{pe}^2 \nu_{en} / \omega_{ce}^2)^{-1}$ [1, 12, 32]. For an electron density of 10^{15} m^{-3} and magnetic field of 0.05 T, we obtain the electron-neutral collision frequency $\nu_{en} \sim 100 \text{ s}^{-1}$ and $\tau_{diff} \sim 65 \text{ ms}$, which is of the order of the expansion times that we measure.

Induced geometric distortions and confinement time

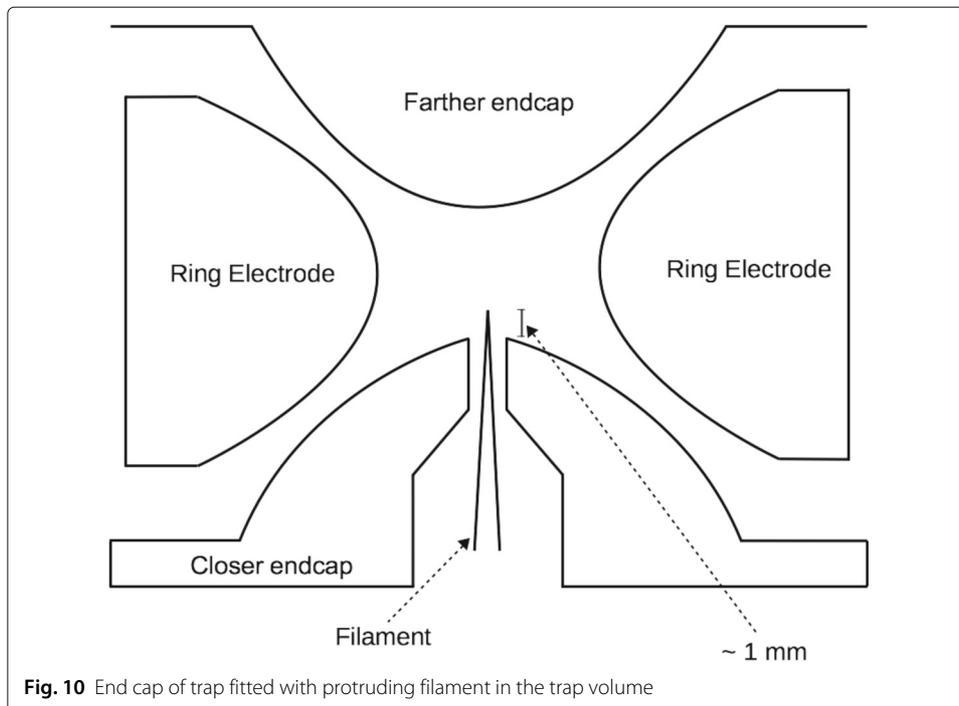
A distortion in the trap geometry is induced by pushing the electron filament tip 1 mm into the volume of the trap, so that it protrudes in as shown in Fig. 10. The confinement time is measured as before for this condition inside the trap:

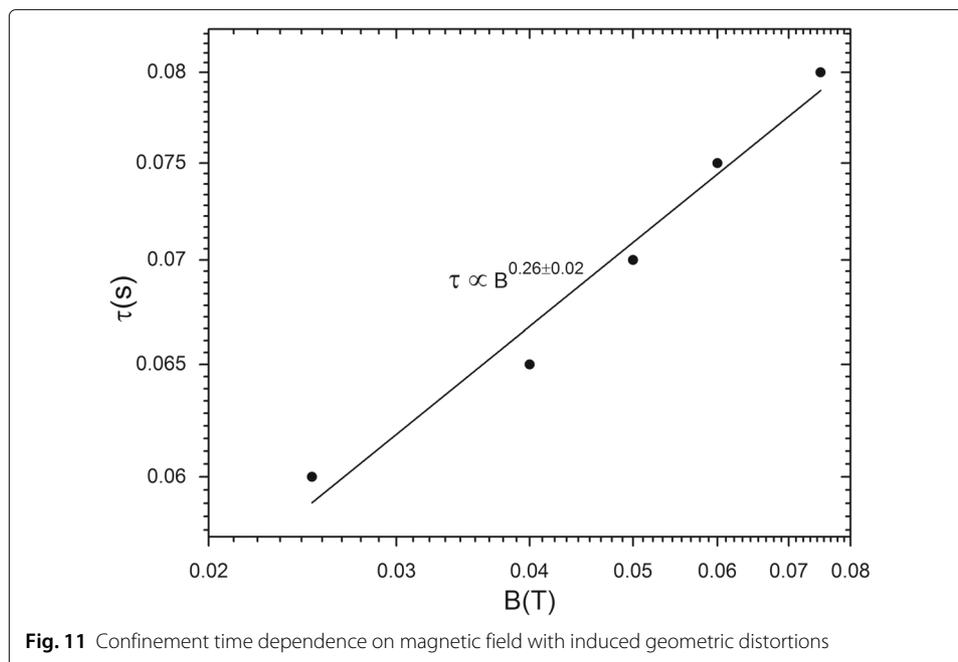
Figure 11 shows a much reduced power dependence on the magnetic field than in the case without the filament tip protruding into the trap. This points to the sensitive dependence of the confinement time on the geometry of the trap. Further distortion of the trap geometry by pushing the filament tip a little more resulted in no confinement for any magnetic field.



Conclusion

We have presented results on measurements of the confinement time of an electron plasma in a quadrupole Penning trap, and the dependence of the same on external magnetic fields. The confinement time departs from the B^2 dependence that is expected for a few particles confined in harmonic traps. Here, an electron plasma of a few eV energy and density of 10^{15} m^{-3} is trapped and this situation enhances space charge effects on





the individual electron's degrees of freedom [5, 33, 35]. For a quadrupole Penning trap, we report a τ dependence $\propto B^{1.41 \pm 0.04}$. Controlled expansion of the plasma through varying the trap potential results in axial expansion of the plasma. The magnetic field dependence on this has been studied as well, and this depends on the kinetic energy range of the confined electrons. We have demonstrated through perturbing the internal geometry of the trap that the confinement time's dependence on the magnetic field reduces significantly and was measured to be $\tau \propto B^{0.26 \pm 0.02}$. Further disturbance of the trap geometry through pushing the filament tip further into the trap resulted in no confinement of the electrons within the magnetic field range used in our experiments, leading us to conclude that the scaling law seems to depend on the nature and extent of the perturbations. Further work to build a theoretical model incorporating higher order perturbative terms in the potential in order to predict the confinement time scaling with B, would help in designing traps that can minimize these perturbative terms [36, 37].

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Authors' contributions

BMD carried out the measurements and wrote the paper. DD helped develop the codes and programmes in LabVIEW. P assisted in the measurements and the data analysis and SA supervised the measurements and wrote the final version of the paper. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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