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Design strategies for controlling damping in micromechanical and nanomechanical resonators

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Abstract

Damping is a critical design parameter for miniaturized mechanical resonators used in microelectromechanical systems (MEMS), nanoelectromechanical systems (NEMS), optomechanical systems, and atomic force microscopy for a large and diverse set of applications ranging from sensing, timing, and signal processing to precision measurements for fundamental studies of materials science and quantum mechanics. This paper presents an overview of recent advances in damping from the viewpoint of device design. The primary goal is to collect and organize methods, tools, and techniques for the rational and effective control of linear damping in miniaturized mechanical resonators. After reviewing some fundamental links between dynamics and dissipation for systems with small linear damping, we explore the space of design and operating parameters for micromechanical and nanomechanical resonators; classify the mechanisms of dissipation into *fluid–structure interactions* (viscous damping, squeezed-film damping, and acoustic radiation), *boundary damping* (stress-wave radiation, microsliding, and viscoelasticity), and *material damping* (thermoelastic damping, dissipation mediated by phonons and electrons, and internal friction due to crystallographic defects); discuss strategies for minimizing each source using a combination of models for dissipation and measurements of material properties; and formulate design principles for low-loss micromechanical and nanomechanical resonators.

Keywords: Dissipation; Damping; Nanomechanical sensing; MEMS; NEMS; Optomechanical systems; Quality factor; Atomic force microscopy; Structural design; Optimization

Introduction

Although damping has been studied for well over a hundred years, the rational design and control of structural damping has seemed a distant goal to many generations of engineers. By way of illustration, we quote from two excellent articles published in the 1990s: “All structures exhibit vibration damping, but despite a large literature on the subject damping remains one of the least well-understood aspects of general vibration analysis” [1], and damping in microcantilevers “is not readily susceptible to engineering analysis” [2].

During the past 15 years, however, there has been a remarkable resurgence of interest in structural damping, especially at small length scales (1 nm to 100 μm), motivated by a host of emerging technologies that include microelectromechanical systems (MEMS), nanoelectromechanical systems (NEMS), nanomechanical sensors, and optomechanical

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systems. This paper presents an overview of major advances in this field from the viewpoint of device design. Our goal is to collect methods and techniques that can provide designers with guidelines and tools for analyzing, controlling, and minimizing damping in miniaturized resonators.

To this end, the next section reviews the fundamental relationships between dissipation and structural dynamics for systems with small linear damping. Subsequently, we explore the space of design and operating parameters for miniaturized mechanical resonators; review the major mechanisms of dissipation by classifying them into three categories: *fluid–structure interactions*, *boundary damping*, and *material damping*; and discuss a set of strategies for controlling each source of damping by combining models for dissipation with measurements of material properties. The concluding section integrates the various strategies in the form of design principles and presents a case-study to illustrate an intriguing trade-off between functionality and performance for bilayered resonators.

Foundations of damping

Let us consider a single mode of oscillations of a linear mechanical resonator. Damping refers to *dissipation* (that is, the conversion of useful mechanical energy into disordered thermal energy) in the oscillating structure, and to the effects of dissipation on *structural dynamics* (see, for instance, [3-9]). Dissipation is quantified using two dimensionless measures: (i) the specific damping capacity, $\Psi = (\Delta W/W)$; and (ii) the loss factor, $\eta = (\Delta W/2\pi W)$. Here, ΔW is the energy dissipated during one cycle of vibration, and W is the maximum elastic strain energy during the vibration cycle. It is difficult to measure energy dissipation or entropy generation directly; hence, damping is estimated by monitoring structural dynamics using such techniques as harmonic excitation, free decay, thermomechanical noise, and wave propagation [5]. Harmonic excitation is associated with two dimensionless measures of damping: the loss angle, ϕ, by which the stress (σ) leads the strain (ε); and the quality factor, $Q \equiv (\omega_n/\Delta\omega)$, where $\Delta\omega$ is the half-power bandwidth of the resonance peak and ω_n is the angular natural frequency. Alternately, the quality factor can be estimated by fitting the resonance peak in the thermomechanical noise spectrum [10-12]. The free decay technique quantifies damping in terms of the logarithmic decrement, δ, and wave propagation techniques measure the attenuation, $\hat{\alpha}$, of the amplitude of elastic waves with wavelength λ [9].

In general, the relationships between the various measures of damping can be quite complicated. However, when damping is linear and small [5,8],

$$\tan\phi = Q^{-1} = \frac{\delta}{\pi} = \frac{\hat{\alpha}\lambda}{\pi} = \frac{\Psi}{2\pi} = \eta = \frac{\Delta W}{2\pi W} \tag{1}$$

Eq. (1) is a cornerstone of the subject of damping. The various relationships are widely used to: (i) quantify damping in miniaturized mechanical structures; (ii) make connections between dissipation and structural dynamics; (iii) compare measurements from different techniques; (iv) compare theoretical predictions with experimental measurements; and (v) design micromechanical and nanomechanical resonators. Indeed, the quality factor is frequently used to estimate damping even when the identities and frequency dependence of the dominant mechanisms of dissipation are not known. In this context, it is worth highlighting the fact that Eq. (1) is *neither exact nor universal*.

Each relationship is an approximation derived from three simple models for linear damping. The first is the classical system consisting of a mass, spring, and viscous dashpot. Viscous damping has its origins in Rayleigh's dissipation function and assumes that the damping force is proportional to the instantaneous velocity of the mass [3]. The second model uses the concept of a complex spring, $k^* = k_1 + i\, k_2$ with $\tan\phi = (k_2/k_1)$. The real and imaginary parts of k^* are called the storage modulus and loss modulus, respectively [9]. The third model is the standard anelastic solid with a constitutive relationship of the form $\sigma + \tau_\sigma \dot{\sigma} = E(\varepsilon + \tau_\varepsilon \dot{\varepsilon})$, where E is the elastic modulus, and τ_σ and τ_ε are material parameters [4,5]. Thus, anelasticity represents an extension of elasticity (as embodied in Hooke's law, $\sigma = E\,\varepsilon$) to incorporate rate effects and relaxation [5]. For these elementary models of viscous, viscoelastic, and anelastic behavior, the approximations in Eq. (1) are within 1% of the exact value for small linear damping with $\phi < 0.01$ [5].

Devices, applications, and design parameters

The first microelectromechanical oscillator was demonstrated in 1967 in the form of a device called the *resonant gate transistor* (RGT) [13]. The RGT consisted of a gold microcantilever (oscillating at 5 kHz with a quality factor of 500) fabricated on a silicon integrated circuit for signal processing applications. Even at this early stage, the benefits of miniaturization for attaining high frequencies and quality factors were well recognized. Soon thereafter, Newell [14] highlighted the value of integrating mechanical structures with microelectronic circuits using microfabrication techniques, and discussed the effects of scale on certain material and structural properties (including fatigue, thermomechanical noise, and viscous air damping). These ideas lay dormant for a decade before exploding into a creative burst of activity that continues unabated to the present day. The invention of the atomic force microscope in 1986 [15], and the rapid evolution of MEMS technologies during the 1990s, provided the motivation and processing capabilities for exploiting miniaturized mechanical resonators for sensing, signal processing, timing, vibration energy harvesting, and precision measurements for fundamental studies in diverse areas of science and engineering [16,17].

In this paper, we focus on one design parameter, namely, the level of damping in miniaturized mechanical resonators. Controlling damping is a ubiquitous and vital requirement because miniaturized resonators require high quality factors ($Q > 10^4$) or ultrahigh quality factors ($Q > 10^6$) for optimal performance [16-19]. The former is typical of MEMS used for signal processing and inertial sensing [16], and the latter of miniaturized resonators used for precision measurements [6]. Unfortunately, controlling damping is a formidable task because numerous mechanisms of dissipation operate in miniaturized resonators. The magnitude of damping is controlled by a large number of variables which include *structural dimensions* (1 nm to 100 μm); *type of structure* (beams, plates, shells, membranes, hollow resonators, fluid-filled micropipes, coupled arrays); *class of material* (metal, alloy, ceramic, glass, elastomer, polymer, nanocomposite); *chemistry* (dopants, impurities); *defects* (type, density, distribution, mobility), *microstructure* (amorphous, polycrystalline, nanocrystalline, single-crystal); *residual stress*; *surface chemistry* and *topography*; *mode of vibration* (flexural, torsional, longitudinal, disc-mode, wineglass-mode, bulk mode); *frequency of oscillation* (100 Hz to 10 GHz); *temperature* (10^{-3} K to 10^3 K); *pressure* (ultrahigh vacuum, atmospheric pressure, immersion in viscous liquid); *amplitude of oscillation*; nature and structure of *boundaries* (structural, thermal, fluidic);

Table 1 Factors that influence damping in miniaturized mechanical resonators

Operation	Structure	Materials	Processing
Mode	Shape	Chemistry	Deposition
Frequency	Size	Alloying	Patterning
Temperature	Architecture	Residual stress	Etching
Pressure	Supporting frame	Microstructure	Bonding
Transduction		Defects	Annealing
		Surfaces	Interconnection
		Interfaces	Packaging

supports, clamps, interconnection and *packaging*; and *processing techniques* used for micromachining and nanofabrication. Table 1 classifies the factors that control damping in miniaturized mechanical resonators. Damping in *oscillators* (that is, a combination of mechanical resonators and an external source of energy for sustaining time-harmonic oscillations) is influenced by an additional set of variables associated with the external source operating in different energy domains [16].

Review of damping

The mechanisms of linear damping can be classified in several different ways. Table 2 shows a scheme based on the spatial origin of dissipation [1]; thus, damping is divided into three categories: *boundary damping*, *fluid–structure interactions* (FSI), and *material damping*. The superscripts indicate the current state of knowledge. The squares (■) denote mechanisms for which detailed models are available for computing dissipation. Hence, design guidelines can be formulated for controlling viscous damping, acoustic radiation, squeezed-film damping, elastic wave radiation, and thermoelastic damping using closed-form expressions or by recourse to numerical analysis using finite-element and finite-difference methods. The section sign (§) denotes mechanisms for which first-order models are available. The challenging task of modeling higher-order phonon-phonon, phonon-surface, phonon-defect, and phonon–electron interactions is an area of current research. Finally, the triangles (▲) denote mechanisms that cannot yet be predicted from first principles. Measurements of material properties are usually required to guide the selection of materials, structures, and processes for controlling internal friction, microsliding, and viscoelasticity.

To gain a qualitative understanding of the various mechanisms, let us consider a generic description of micromechanical and nanomechanical resonators. Typically, the resonators are attached to relatively large *supporting frames* to facilitate handling and

Table 2 Classification of linear damping mechanisms

Boundary damping		Fluid–structure interactions		Material damping	
Elastic wave radiation■	[20-39]	Viscous damping■	[40-42]	Thermoelastic damping■	[43-59]
Microsliding▲	[60-63]	Squeeze-film damping■	[16,64]	Phonon-phonon interactions§	[65-67]
Viscoelasticity▲	[9]	Acoustic radiation■	[68]	Phonon–electron interactions§	[6,65]
		Internal flow■	[69,70]	Internal Friction▲	[4,5,71-84]

interconnection, and then *packaged* by mounting the supporting frame inside test stations, instruments, or customized packages. Illustrative examples include commercial silicon microcantilevers (1 to 10 μm thick and 0.1 to 0.5 mm long) used for atomic force microscopy (AFM), and silicon nitride nanomembranes (30 to 200 nm thick with lateral dimensions ranging from 10 μm to 1 mm) used in optomechanical systems [85,86]. In both cases, the devices are attached to silicon frames with thickness of a few hundred microns and lateral dimensions of a few millimeters. The attachment can be monolithic (exemplified by bulk-micromachined silicon resonators) or engendered by the adhesion of thin films to the supporting frame in the case of surface-micromachined devices [16].

The materials, methods, and designs used for packaging vary widely depending on the device and application. For example, silicon microcantilevers are loaded into AFMs using clamps, spring-loaded clips, or adhesive gels, but other devices require custom-designed packages for electrical connections, thermal interfaces, microfluidic manifolds, or vacuum packaging to maintain the resonator at low pressure [16]. The dissipation due to all components associated with the supporting frame and package is collectively called boundary damping. There are three main mechanisms of boundary damping: (i) *support losses* or *anchor losses* due to the radiation of elastic waves (or stress waves) from the resonator into the supporting frame [20-35]; (ii) *microsliding* at the interfaces between the resonator and supporting frame, and between the supporting frame and package [61]; and (iii) *viscoelasticity* in the gels and adhesives used to bond the supporting frame to the package [9]. (The term *clamping loss* is frequently encountered in the literature. Depending upon the context, it can refer either to stress-wave radiation or to microsliding).

The application dictates the fluidic environment of the resonator. When a miniaturized mechanical structure oscillates in a chamber containing a gas or liquid, energy is dissipated during every cycle of vibration due to the conversion of ordered structural energy into the thermal energy of the molecules of the fluid. Fluid–structure interactions (FSI) have been studied extensively using experiment, theory, and computation because they are a major source of damping. Indeed, the immense literature on FSI in microscale and nanoscale resonators deserves a dedicated full-fledged review in its own right. A detailed understanding of viscous damping, squeezed-film damping, and acoustic radiation has been obtained for numerous devices including plates, membranes, beams, torsional resonators, and hollow resonators containing internal microfluidic channels. The dependence of damping on material properties, fluidic properties, geometry, size, confinement, mode, frequency, pressure, and temperature has been captured well by models (see, for example, [14,16,40-42,64,68-70]).

Material damping refers to all dissipative mechanisms that operate within the volume (*bulk*), at the *free surfaces*, and at the internal *interfaces* of the resonator. Composite architectures are ubiquitous in MEMS and NEMS technologies; hence, several types of interfaces (layer boundaries, grain boundaries, and phase boundaries) can be encountered in miniaturized resonators. The creation of interfaces is often unintentional and, in some cases, undesirable. For example, the free surfaces of silicon, titanium, and aluminum are invariably covered with ultrathin coatings of native oxide under typical processing and operating conditions. Similarly, internal oxide layers and interfaces can be created due to unintentional oxidation during deposition [87].

Material damping can be divided into two categories: *fundamental damping* and *internal friction* [6]. The former is a set of mechanisms (thermoelastic damping, phonon-phonon interactions, and phonon–electron interactions) that set the ultimate lower limit on damping. These mechanisms have their origin in the electronic, atomic, and molecular structure of materials, and operate even in the idealized limit of perfectly engineered devices (for example, high-quality single-crystal materials with negligible defect density). Internal friction refers to damping caused by the irreversible motion of crystallographic defects (for example, vacancies, divacancies, interstitial atoms, substitutional atoms, surface adatoms, edge dislocations, screw dislocations, grain boundaries, phase boundaries, layer boundaries, and precipitates) [4], and each type of defect can give rise to several mechanisms of dissipation [5].

The standard anelastic solid provides a conceptually simple picture of material dissipation as a relaxation process. When damping is governed by a single relaxation time τ, the frequency dependence of material damping is a Debye peak given by [4]

$$Q_{\mathrm{material}}^{-1} = \Delta \frac{\omega \tau}{1 + (\omega \tau)^2} \tag{2}$$

where Δ is called the relaxation strength. Table 3 lists the relaxation strength and time for the two main sources of fundamental dissipation–*thermoelastic damping* due to irreversible heat conduction and *Akhiezer damping* due to phonon-phonon interactions–for an isotropic, homogeneous beam of thickness h. In this table, E is the Young's modulus, ρ is the density, α is the coefficient of thermal expansion, T is the absolute temperature, C is the specific heat per unit volume, k is the thermal conductivity, v is the velocity of sound, and γ is Grüneisen's constant. A preliminary estimate of fundamental losses can be obtained using the nominal properties listed in Table 4 for common metals and ceramics used in MEMS and NEMS. For detailed calculations, it is necessary to account for changes in properties due to size effects, microstructure, crystallographic anisotropy, and processing conditions.

Techniques for measuring internal friction in thin films were developed over 30 years ago [72], but many classes of materials, structures, and defects still remain largely unexplored. The sparse literature on this topic can be classified into two broad categories. The first set focuses on the effects of temperature on dissipation in an effort to measure relaxation peaks and study the underlying defect interactions (see, for example, [72-77]). The second set explores the effects of processing conditions (including post-deposition annealing), residual stress, and frequency on internal friction (see, for example, [78-83]). In general, the dissipation spectra rarely exhibit the Debye peak predicted by Eq. (2); instead, internal friction is often a weak monotonic function of frequency [9]. Explanations for this behavior range from a distribution of activation energies and relaxation times [5,9], to a hierarchically constrained sequence of serial relaxation processes in which the fast degrees of freedom (involving the motion of single atoms) must relax

Table 3 Simple expressions for estimating fundamental material losses using Eq. (2) [5,6]

Mechanism	Relaxation strength	Relaxation time
Thermoelastic damping (TED)	$\Delta_{\mathrm{TED}} = \frac{E\alpha^2 T}{C}$	$\tau_{\mathrm{TED}} = \frac{C h^2}{\pi^2 k}$
Akhiezer damping	$\Delta_{\mathrm{Akhiezer}} = \frac{C T \gamma^2}{\rho \mathrm{v}^2}$	$\tau_{\mathrm{Akhiezer}} = \frac{3k}{C \mathrm{v}^2}$

Table 4 Nominal properties of common metals and ceramics at 300 K [98]

Material	E (GPa)	ρ (kg/m^3)	k (W/m/K)	α (K^{-1})	C (J/m^3/K)	v (m/s)	γ
Aluminum	70	2.7×10^3	220	24.0×10^{-6}	2.4×10^6	5.1×10^3	1.7
Copper	120	8.9×10^3	400	20.0×10^{-6}	3.8×10^6	3.6×10^3	1.9
Gold	82	1.9×10^4	320	15.0×10^{-6}	2.5×10^6	2.1×10^3	2.9
Nickel	210	8.9×10^3	92	13.0×10^{-6}	3.9×10^6	4.8×10^3	1.8
Silicon	160	2.3×10^3	150	2.6×10^{-6}	1.6×10^6	8.3×10^3	0.4
Silicon carbide	400	3.2×10^3	70	3.0×10^{-6}	3.0×10^6	1.1×10^4	1.1
Silicon oxide	70	2.2×10^3	1	0.5×10^{-6}	1.5×10^6	5.6×10^3	1.9
Silicon nitride	250	3.2×10^3	8	3.0×10^{-6}	3.0×10^6	8.8×10^3	1.2
Silver	76	1.1×10^4	430	18.0×10^{-6}	3.0×10^6	2.7×10^3	2.4

before the slower degrees of freedom involving the coordinated motion of groups of atoms [84].

Strategies for controlling damping

Fluid–structure interactions (FSI)

Fluid–structure interactions typically limit the quality factor of miniaturized mechanical resonators to low values ranging from 1 to 10^3. The most effective strategy for eliminating FSI is to operate the device at low pressure. For values of the Knudsen number (Kn) below 0.01, the gas is effectively a viscous continuum and damping is independent of pressure [14]. As the pressure is reduced, the flow transitions to the free molecular regime (FMR) for Kn > 10, and the damping is given by [14]

$$Q_{\mathrm{FMR}}^{-1} = \sqrt{\frac{32M}{\pi R T}} \frac{P}{h \rho \omega} \tag{3}$$

where M is the molar weight of the gas, R is the universal gas constant, and P is the gas pressure. When the pressure is reduced further, fluidic damping becomes negligible at a critical value which is a function of the size, shape, and mode of the resonator. The critical pressure has been measured to range from 0.1 Pa to 10^3 Pa for miniaturized mechanical resonators [14,29,40-42].

Elastic wave radiation

Elastic waves are generated when the resonator applies time-harmonic forces and moments at the point of attachment to the support. This phenomenon is effectively a source of damping because the ordered mechanical energy of the resonator is transferred to the substrate in the form of stress waves, and eventually dissipated in the substrate. By assuming that the radiated energy does not reflect back into the resonator, damping can be estimated by performing calculations solely in the elastic domain without specifying the detailed mechanisms by which energy is dissipated in the substrate [20-25]. One of the first models for support loss is an analysis by Jimbo and Itao in 1968 of a two-dimensional system consisting of an isotropic, homogeneous, linear elastic cantilever of length L, width w, and thickness h that is attached to an elastic half-space. For this idealized geometry, the support loss is proportional to $(h/L)^3$ [20]. When

the finite size of the supporting frame is taken into account, support losses in typical bulk-micromachined cantilevers are given by

$$\begin{aligned} Q^{-1}_{\text{support}} &= 0.95\frac{w}{L}\frac{h^2}{h_s^2} \qquad (\text{for } h_s \ll \lambda) \\ Q^{-1}_{\text{support}} &= 0.31\frac{w}{L}\left(\frac{h}{L}\right)^4 \qquad (\text{for } h_s \gg \lambda) \end{aligned} \tag{4}$$

where h_s is the thickness of the support and λ is the wavelength of the elastic wave propagating in the support [22].

Support losses can be reduced by using analytical and numerical models for stress-wave radiation to guide the selection of materials, structures, dimensions, modes, and frequencies. Alternately, the generation and propagation of elastic waves can be disrupted by contacting the resonator at its nodal points using anchors or tethers [26-31] and incorporating phononic band-gap structures [32-35], acoustic reflectors [36,37], and vibration isolators [38,39]. In each case, well-established models from vibrations and elasticity are available to guide design.

Microsliding and viscoelasticity

The other sources of boundary damping face challenges that are common to many aspects of thin-film adhesion and packaging of MEMS [16]. The variety of designs, processes, techniques, and materials makes it difficult to develop general guidelines for a large class of devices and applications. Models for adhesion and friction can provide useful qualitative insights into the underlying mechanisms but improving thin-film adhesion and reducing microsliding remains an art. Nevertheless, several general strategies can be formulated: (i) minimize deformations and strains at the interface between the supporting frame and package: for example, by employing the anti-symmetric modes of dual-cantilever beams [60] or double-paddle oscillators [62,63], and acoustically isolating the resonator from the supporting frame and package; (ii) use precision-machined clamps and avoid spring-loaded clips, gels, polymer-based adhesives, and die bonds for packaging; and (iii) ensure good adhesion between the resonator and supporting frame by activating the substrate before depositing structural thin films, employing adhesion promoters in the form of ultrathin films of Ti or Cr, and using ion-beam assisted deposition techniques [88].

Thermoelastic damping

Thermoelastic damping (TED) is a rare example of a dissipative mechanism that can be computed accurately from first principles [4]. Consider a beam or a plate that is subjected to time-harmonic bending forces or moments. The elastic stresses in the structure give rise to elastic strains (which are in phase with the stress) and thermal strains due to thermoelastic coupling. In general, the thermal strains are not in phase with the driving elastic stresses; hence, energy is dissipated during every cycle of vibration. Alternately, TED can be viewed as the result of oscillating temperature gradients generated by oscillating stress gradients in thermoelastic solids. Heat conduction across finite temperature gradients leads to entropy generation and energy dissipation [43-48].

The analysis of TED is conceptually straightforward for heat conduction in the diffusive regime (that is, the mean free path of thermal phonons is much less than the

characteristic length scale of the resonator [46]). The formula in Table 3 is due to a celebrated analysis by Zener in 1937. More recently, Zener's analysis of homogenous, isotropic beams has been extended in multiple directions and there are now over 350 significant publications on this topic. A selection of the literature includes models to account for the effects of structural boundaries [47,48]; polycrystalline grain structure [49,50]; layered composite architecture [51-53]; electrostatic actuation [54]; and geometry (plates [54,55], slotted, channeled, and hollow beams [56-58], double-paddle oscillators [63], and bulk-mode, ring-mode, and disc-mode resonators [59]). Using these models, the design space can be explored to formulate detailed guidelines for selecting geometries, structures, modes, materials, and frequencies to minimize thermoelastic damping.

Internal friction

The magnitude of internal friction is governed by the type, distribution, density, and mobility of defects, and by the interactions between different classes of defects. Quantifying these details, especially in micrometer and nanometer scale structures, requires extensive experimental studies using a suite of microscopic and spectroscopic techniques (see, for example, [82,87,89]). Even when such information is available, it is difficult to model the dynamics of defects over multiple scales of length, time, and energy. Therefore, in common with many other material properties, design guidelines are derived by measuring internal friction, characterizing the microstructure, and formulating *process-structure–property relationships* [5,9,71].

There are three main strategies for controlling dissipation due to internal friction. The first is to control the type, distribution, and density of defects. High-quality single-crystals can be used for resonators but fabrication challenges currently limit this option to a small set of materials (for example, commercial silicon wafers and epitaxial thin films grown using molecular beam epitaxy). More practically, defects can be controlled by an appropriate selection of the deposition technique, processing conditions, and post-deposition heat treatment. For example, when sputter-deposited aluminum films were annealed at 450°C for 1 hour in an inert atmosphere, the average grain size increased from 100 nm to 390 nm, and the room-temperature values of internal friction reduced from 0.05 to 0.02 [82].

The second strategy is to reduce the mobility of defects by materials selection and materials design. Defect mobility is a function of several variables including atomic bonding, crystal structure, microstructure, melting temperature, operating temperature, and frequency. The lattice self-diffusivity at the melting point is $\sim10^{-16}$ m^2/s for covalently bonded diamond-cubic materials, $\sim5\times10^{-14}$ m^2/s for oxides, and $\sim5\times10^{-13}$ m^2/s for face-centered cubic metals [90]. All else being equal, defects are less mobile in brittle ceramics (Si, SiO_2, quartz, SiC, TiO_2, Al_2O_3, and Ta_2O_5) than in common metals and alloys [6]. Thus, for precision measurements requiring ultrahigh quality factors, multilayer stacks of dielectric films consisting of alternate layers of amorphous silica (SiO_2) and amorphous tantala (Ta_2O_5) are preferred over metallic thin films for optical coatings [17]. Furthermore, small quantities of alloying additions can have a disproportionately large influence on defect mobility. For instance, damping in aluminum alloy Al 5056 is an order of magnitude less than that in pure aluminum over a broad range of temperatures and frequencies [6,74].

Finally, internal friction can be altered by changing the operating temperature but the implementation of this strategy must take into account the non-monotonic behavior of dissipation. For example, internal friction in bulk quartz crystals increased by over two orders of magnitude (from 10^{-7} to 4×10^{-5}) upon cooling from 300 K to 50 K, and then reduced precipitously to a remarkably low value of 2×10^{-10} when cooled further to 2 K [91].

Stress-engineered resonators

Micromechanical and nanomechanical resonators are commonly subjected to a pre-stress (or residual stress) originating from intrinsic stresses generated during the growth of thin film materials [92] and differential expansion caused by thermal excursions during processing [16]. The pre-stress can affect several properties (stiffness, natural frequencies, mode shapes, linearity, and damping) and reliability (fracture under tension, buckling under compression, inelastic deformation, and stress relaxation by creep [93]). Therefore, the control and mitigation of residual stresses is a major consideration in the design of surface-micromachining processes [16].

In some cases, however, large tensile residual stresses can be used to reduce damping, as exemplified by high-Q devices fabricated using amorphous silicon nitride films grown by low-pressure chemical vapor deposition (see, for example, [83,86,94-96]). Indeed, quality factors of 50 million have been obtained at room temperature with highly-stressed (~1 GPa) nanomembrane resonators [94]. In general, the effects of the tensile pre-stress are governed by the relative magnitudes of two factors: (i) increase in the elastic stored energy (both extensional and flexural), and (ii) stress-induced changes in dissipation. The former can be obtained by analyzing the elastic deformation of the resonator (see, for example, [95-97] for expressions of the quality factors for stretched-string resonators), but the latter has been analyzed explicitly only for a few mechanisms. Notable examples include thermoelastic damping in plates [54] and membranes [94]; in both cases, the magnitude of TED can be reduced by applying an in-plane tensile pre-stress.

Design principles

Miniaturized mechanical resonators are enabling a remarkably large and diverse set of applications ranging from sensing, timing, and signal processing to scanning probe microscopy and precision measurements for fundamental studies in several fields of engineering and science. New concepts continue to emerge; devices are growing in sophistication, performance, and functionality; and there is a growing emphasis on transitioning from proofs-of-concept to full-fledged commercialization. All these trends have created an urgent need for developing effective and rational methods for design, analysis, and optimization.

When microresonator technologies began to emerge in the 1980s, damping was controlled using iterative cycles of fabrication, testing, and analysis at the device level [2]. Unfortunately, the design space is much too vast and complex to be probed efficiently in this manner, especially considering the time and resources required for integrating new materials and structures into micromachining and nanofabrication process flows [16]. It is therefore useful and timely to develop methods that account for damping at every stage of the design cycle. In this section, we present an approach that is based on identifying the mechanisms of damping, and then developing strategies for controlling

each mechanism with a judicious combination of models for dissipation and measurements of material properties.

Using the standard concept-embodiment-detail description of the design process [98], consider a typical problem that begins by translating market needs and application requirements into a set of device specifications, which are then used to generate a set of preliminary designs. At this stage, a careful estimation of fundamental material losses can establish the ultimate limits of dissipation and serve as a criterion for ranking the various designs. The device can approach the ultimate limits only to the extent that all other sources of dissipation (FSI, boundary damping, and internal friction) are eliminated. The optimal solutions are to: (i) eliminate FSI by operating at a sufficiently low pressure (ranging from 0.1 Pa to 10^3 Pa, depending on the size, shape, and mode of the resonator); (ii) eliminate boundary damping by decoupling the resonator from the supports and package by using nodal supports, phononic bandgaps, and acoustic isolators; and (iii) eliminate internal friction by using high-quality single-crystals or materials with low defect mobility. In the literature, we can now find a small, but steadily growing, set of devices that can approach the ultimate limits of damping (see, for example, [6,17,28,63,80]).

In many cases, however, the optimal solutions cannot be implemented because the design is constrained by the application, functionality, and actuation, and by the limitations of processing and packaging technologies. As an example, consider the class of bilayered metal-coated ceramic resonators that are widely used in MEMS and NEMS. The ceramic structure can be designed to oscillate with low damping approaching the fundamental limits. The coating adds valuable functionality by enhancing the optical reflectivity and electrical conductivity, but severely degrades the quality factor and performance of the device due to the relatively large internal friction in polycrystalline metallic films. Thus, the design problem is to identify an optimal trade-off between performance and functionality.

When dissipation is dominated by internal friction (IF) in the metallic film, Eq. (1) can be used to express the inverse quality factor of the bilayer as

$$Q^{-1}_{\text{bilayer}} = \frac{\Delta W_{\text{bilayer}}}{2\pi W_{\text{bilayer}}} \simeq \frac{\Delta W_{\text{IF, film}}}{2\pi W_{\text{bilayer}}} = \left(\frac{W_{\text{film}}}{W_{\text{bilayer}}}\right)\frac{\Delta W_{\text{IF, film}}}{2\pi W_{\text{film}}} = \left(\frac{W_{\text{film}}}{W_{\text{bilayer}}}\right) Q^{-1}_{\text{IF, film}} \qquad (5)$$

where $(W_{\text{film}}/W_{\text{bilayer}})$ is obtained by computing the elastic deformation of the film and bilayer. Eq. (5) suggests two distinct strategies for increasing the quality factor. The first is to control internal friction in the film using process-structure–property relationships. For sputtered aluminum films (which are widely used as coatings for numerous applications including commercial probes for atomic force microscopy), internal friction can be reduced by post-deposition annealing to increase grain size [82], reducing film thickness [80], and alloying aluminum with small amounts (5%) of magnesium [6,74]. Alternately, aluminum can be replaced with gold [80] or multilayer dielectric stacks (for example, alternate layers of silica and tantala [17]). The second strategy is to minimize the ratio of elastic strain energies by confining the metallic coating to regions of low deformation and strain [99,100]. For microcantilevers oscillating in the fundamental bending mode, internal friction due to metallization can be made negligible by coating only the tip of the beam [99].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
SJ analyzed the relationships between dissipation and dynamics, formulated the case study, and contributed to the sections on fluid–structure interactions. SH contributed to the sections on boundary damping. SV contributed to the sections on material damping and design methodologies. The manuscript was edited by SV, and then read and approved by all authors.

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